[E.N / 15] **ARAB REPUBLIC OF EGYPT** [٥٤ / ج] ث.ع / ث / ح

Ministry of Education

General Secondary Education Certificate Examination, 2015

[New System – Second Session]

Pure Mathematics [Algebra & Solid Geometry] **Time: 2 hours**

الرياضيات البحتة [الجبر والهندسة الفراغية] باللغة الإنجليزية

تنبيه مهم : ١ - يسلم الطالب ورقة امتحانية باللغة العربية مع الورقة المترجمة .
٢ - الإجابات المتكررة عن أسئلة الاختيار من متعدد لن تقدر ويتم تقدير الإجابة الأولى فقط .

Remark: 1- Calculators are allowed. 2- $\{1, \omega, \omega^2\}$ are the cubic roots of unity and $i^2 = -1$

[الأسئلة فى صفحتين]

First: Answer only One question of the following:

First Question: Complete the following statements: (6 marks)

a) If ${}^{25}C_{2r-14} = {}^{25}C_{r-1}$, then the value of r = ...

b) The system of equations: $kx + y + 3z = 9$, $2x + ky + 2z = 2$ and $x + 2y + z = 1$ has no solution if k = ...

c) If a line not belonging to a plane is parallel to a line in the plane, then it is .....

d) ABCD is a square of side length 8 cm. $\overline{AM}$ is drawn perpendicular to the plane of the square. If $AM = 8\sqrt{3}$ cm, then the measure of the angle between $\overline{MB}$ and the plane ABCD equals ..... °

e) If the sum of lengths of the diagonals of a cube is $24\sqrt{3}$ cm, then the area of one of its faces equals ..... cm^2.

f) MABCD is a right quadrilateral pyramid, the length of its lateral altitude is 5 cm. If the area of its base is 36 cm^2, then its height equals ..... cm.

Second Question: Choose the correct answer from those given: (6 marks)

a) If ${}^{9}P_r : {}^{9}P_{r+1} = 1 : 7$, then r = ... [2 or 3 or 4 or 5]

b) If $1, \omega, \omega^2$ are the cubic roots of the unity, $n \in \mathbb{Z}^+$, then

$$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$$ equals ..... [1 or ω or ω^2 or 0]

c) If MABCD is a right quadrilateral pyramid, then the line of intersection of the two planes MBC, MAD is .....

[$\overleftrightarrow{AD}$ or $\overleftrightarrow{MB}$ or a line passing by M parallel to $\overline{BC}$ or a line passing by M parallel to $\overline{AB}$]

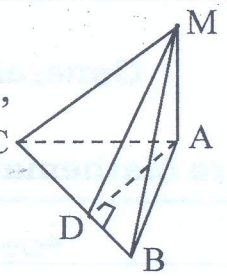
d) If the sum of areas of the faces of a regular triangular pyramid = $100\sqrt{3}$ cm^2, then the sum of lengths of its edges equals ..... cm

[60 or $60\sqrt{2}$ or $60\sqrt{3}$ or 120]

[بقية الأسئلة فى الصفحة الثانية]

e) In the opposite figure:

MABC is a triangular pyramid. If $\overline{MA} \perp$ plane ABC, $\overline{AD} \perp \overline{BC}$, then $\overline{BC} \perp$ .....



[$\overline{MB}$ or $\overline{MC}$ or $\overline{MD}$ or $\overline{AC}$]

f) In the previous figure:

If $m(\angle M - \overline{BC} - A) = 30^\circ$, then .....

[$MA = \sqrt{3}\,AD$ or $AD = \sqrt{3}\,MA$ or $MA = AD$ or $MA = \frac{1}{2}\,AD$]

(Second) Answer the following Questions:

Third Question: (8 marks)

a) (i) If $n \mid n+2 = 60 \mid n$, find the value of n.

(ii) Prove that: $(a + b\omega + a\omega^2)(a + b\omega^2 + a\omega^4) = (b - a)^2$

b) Solve the following equations using Cramer's rule:

$2x + 3y - z = 0$, $x - 3y + 2z = 2$, $y + z = -2$

Fourth Question: (8marks)

a) Find the two square roots of the number $z = -3 + 4i$ without transforming to the trigonometric form.

b) Without expanding the determinant, prove that:

$$\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{26}+\sqrt{15} & 5 & \sqrt{10} \\ \sqrt{65}+3 & \sqrt{15} & 5 \end{vmatrix} = 5\sqrt{3}\,(\sqrt{6}-5)$$

Fifth Question: (8 marks)

a) X and Y are two parallel planes, M is a point not belonging to any of them. The lines $\overleftrightarrow{MA}$, $\overleftrightarrow{MC}$, $\overleftrightarrow{ME}$ are drawn to intersect plane X at A, C, E and plane Y at B, D, F respectively. If $\frac{MA}{MB} = \frac{2}{5}$ and AC = 4 cm, CE = 3 cm, AE = 5 cm, find the perimeter of Δ BDF.

b) ABC is a triangle in which AB = AC = 13 cm, BC = 10 cm, M is the mid-point of $\overline{BC}$. $\overline{AL}$ is drawn perpendicular to the plane of the triangle so that AL = 12 cm.

(i) Calculate the length of $\overline{AM}$ and prove that $\overline{ML} \perp \overline{BC}$

(ii) Find $m(\angle A - \overleftrightarrow{BC} - L)$

(iii) Prove that the plane LAM $\perp$ the plane LBC

[انتهت الأسئلة]

الدرجة العظمى (٣٠)	جمهورية مصر العربية وزارة التربية والتعليم امتحان شهادة إتمام الدراسة الثانوية العامة لعام ٢٠١٥ م نموذج إجابة [الرياضيات البحتة " الجبر والهندسة الفراغية بالإنجليزية "]	[٥٤] الدور الثانى (نظام حديث)
الدرجة الصغرى (–)		
عدد الصفحات(٥)		

Question (1): (6 Marks): One Mark for each item

(a) 13 **1**

(b) 3 or 4 **1**

(c) Parallel to the plane **1**

(d) 60 **1**

(e) 36 **1**

(f) 4 **1**

(تراعى الحلول الأخرى)

Question (2): (6 Marks): One Mark for each item

(a) 2 **1**

(b) Zero **1**

(c) a line passing by M parallel to $\overline{BC}$ **1**

(d) 60 **1**

(e) $\overline{MD}$ **1**

(f) $AD = \sqrt{3}MA$ **1**

(تراعى الحلول الأخرى)

Question (3): (8 Marks): (a) 4 Marks, (b) 4 Marks

(a) (i) $\because n\lfloor n+2 = 60\lfloor n$

$\therefore n(n+1)(n+2)\lfloor n = 60\lfloor n$ **0.5**

$\therefore {}^{n+2}P_3 = 5\times4\times3 = {}^{5}P_3$ **1**

$\therefore n+2 = 5 \implies n = 3$ **0.5**

(ii) $l.h.s. = [a(1+\omega^2)+b\omega]\ [a(1+\omega)+b\omega^2]$ **0.5**

$= [b\omega - a\omega]\ [b\omega^2 - a\omega^2]$ **0.5**

$= \omega^3\ (b-a)\ (b-a)$ **0.5**

$= (b-a)^2$ **0.5**

(b) $\Delta = \begin{vmatrix} 2 & 3 & -1 \\ 1 & -3 & 2 \\ 0 & 1 & 1 \end{vmatrix} = -14$ **1**

$\Delta_1 = \begin{vmatrix} 0 & 3 & -1 \\ 2 & -3 & 2 \\ -2 & 1 & 1 \end{vmatrix} = -14$ **0.5**

$\Delta_2 = \begin{vmatrix} 2 & 0 & -1 \\ 1 & 2 & 2 \\ 0 & -2 & 1 \end{vmatrix} = 14$ **0.5**

$\Delta_3 = \begin{vmatrix} 2 & 3 & 0 \\ 1 & -3 & 2 \\ 0 & 1 & -2 \end{vmatrix} = 14$ **0.5**

$\therefore x = \frac{\Delta_1}{\Delta} = 1$ **0.5** $, y = \frac{\Delta_2}{\Delta} = -1$ **0.5** $, z = \frac{\Delta_3}{\Delta} = -1$ **0.5**

$\therefore S.S. = \{(1,-1,-1)\}$

(تراعى الحلول الأخرى)

Question (4): (8 Marks): (a) 4 Marks, (b) 4 Marks

(a) Let $\sqrt{-3+4i} = x + iy$

$\therefore (x+iy)^2 = -3+4i$ **0.5**

$\therefore x^2 - y^2 + 2xy\,i = -3+4i$

$x^2 - y^2 = -3$ ..... (1) **0.5**

$2xy = 4$ ..... (2) **0.5**

$x^2 + y^2 = 5$ ..... (3) **0.5**

Adding (1), (3) $\therefore 2x^2 = 2 \implies x^2 = 1$

$\therefore x = \pm 1$ **0.5**

From (1), (3) by subtraction $\therefore 2y^2 = 8 \implies y^2 = 4$

$\therefore y = \pm 2$ **0.5**

$\because xy > 0$ $\therefore x, y$ have the same sign

$\therefore \sqrt{-3+4i} = \pm(1+2i)$ **1**

(b) $\Delta = \begin{vmatrix} \sqrt{13} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{26} & 5 & \sqrt{10} \\ \sqrt{65} & \sqrt{15} & 5 \end{vmatrix} + \begin{vmatrix} \sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15} & 5 & \sqrt{10} \\ 3 & \sqrt{15} & 5 \end{vmatrix}$ **1**

$= \sqrt{13}\times\sqrt{5}\times\sqrt{5}\begin{vmatrix} 1 & 2 & 1 \\ \sqrt{2} & \sqrt{5} & \sqrt{2} \\ \sqrt{5} & \sqrt{3} & \sqrt{5} \end{vmatrix}$ **0.5** $+ \sqrt{3}\times\sqrt{5}\times\sqrt{5}\begin{vmatrix} 1 & 2 & 1 \\ \sqrt{5} & \sqrt{5} & \sqrt{2} \\ \sqrt{3} & \sqrt{3} & \sqrt{5} \end{vmatrix}$ **0.5** $C_1 \to C_1 - C_2$

$= 0 + 5\sqrt{3}\begin{vmatrix} -1 & 2 & 1 \\ 0 & \sqrt{5} & \sqrt{2} \\ 0 & \sqrt{3} & \sqrt{5} \end{vmatrix}$ **1** $R_3 \to R_3 - \frac{\sqrt{3}}{\sqrt{5}}R_2$

$= 5\sqrt{3}\begin{vmatrix} -1 & 2 & 1 \\ 0 & \sqrt{5} & \sqrt{2} \\ 0 & 0 & \sqrt{5} - \frac{\sqrt{6}}{\sqrt{5}} \end{vmatrix}$ **0.5**

$= -5\sqrt{3}\times\sqrt{5}\left(\sqrt{5} - \frac{\sqrt{6}}{\sqrt{5}}\right)$

$= 5\sqrt{3}\ (\sqrt{6} - 5)$ **0.5**

(تراعى الحلول الأخرى)

Question (5): (8 Marks): (a) 4 Marks, (b) 4 Marks

a) ∵ plane X // plane Y

and plane MBD intersects them

$\therefore \overline{AC} // \overline{BD}$

$\therefore \frac{AC}{BD} = \frac{MA}{MB} = \frac{2}{5}$ ..... (1) **1**

Similarly $\overline{CE} // \overline{DF}$

$\therefore \frac{CE}{DF} = \frac{MC}{MD} = \frac{MA}{MB} = \frac{2}{5}$ ..... (2)

Also $\overline{AE} // \overline{BF}$

$\therefore \frac{AE}{BF} = \frac{MA}{MB} = \frac{2}{5}$ ..... (3) **0.5**

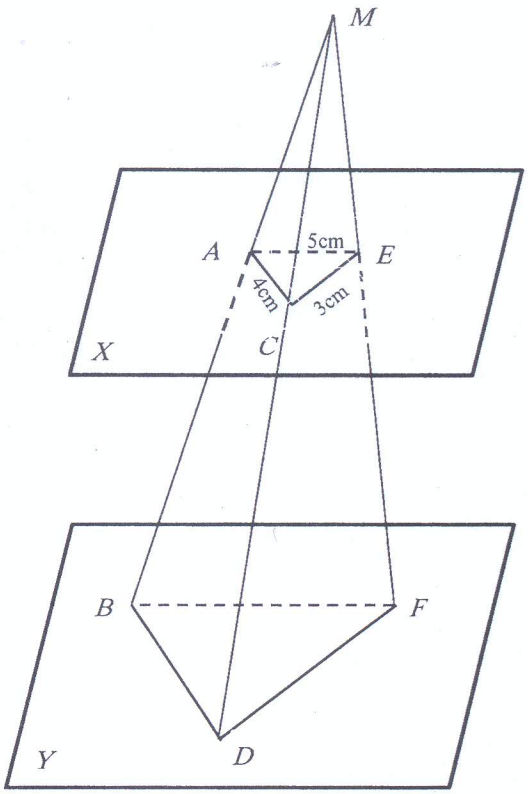
From (1), (2), (3) $\therefore \frac{AC}{BD} = \frac{CE}{DF} = \frac{AE}{BF} = \frac{2}{5}$

$\therefore \Delta ACE \sim \Delta BDF$ **0.5**

$\therefore \frac{\text{Per}(\Delta ACE)}{\text{Per}(\Delta BDF)} = \frac{2}{5}$ **0.5**

$\therefore \frac{12}{\text{Per}(\Delta BDF)} = \frac{2}{5}$

$\therefore \text{Per}(\Delta BDF) = \frac{5 \times 12}{2} = 30\ cm$ **1**



0.5 for drawing

b) (i) ∵ $AB = AC$, M is mid-point of $\overline{BC}$

$\therefore \overline{AM} \perp \overline{BC}$ $\therefore AM = \sqrt{(13)^2 - (5)^2}$

$\therefore AM = 12\ cm$ **0.5**

∵ $\overline{LA} \perp$ plane ABC, $\overline{LM}$ is inclined, its projection $\overline{AM} \perp \overline{BC}$

$\therefore \overline{LM} \perp \overline{BC}$ **0.5**

(ii) $\therefore \angle LMA$ is a plane angle of $\angle A - \overleftrightarrow{BC} - L$ **0.5**

$\therefore \tan\theta = \frac{LA}{AM}$ **0.5**

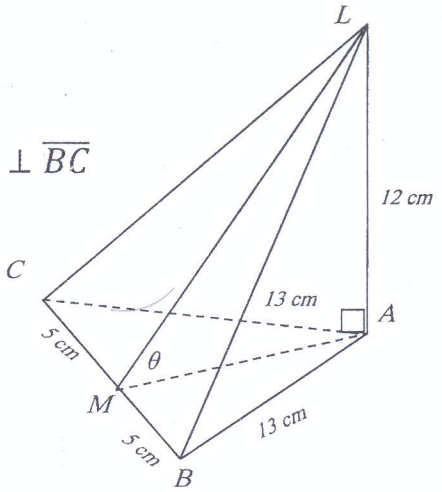
$\therefore \tan\theta = \frac{12}{12} = 1$

$\therefore \theta = 45°$ **0.5**

(iii) ∵ $\overline{BC} \perp$ both $\overline{AM}, \overline{LM}$

$\therefore \overline{BC} \perp$ plane LAM **0.5**

∵ $\overline{BC} \subset$ plane LBC $\therefore$ plane $LBC \perp$ plane LAM **0.5**



0.5 for drawing

(تراعى الحلول الأخرى)

انتهى نموذج الإجابة